Beyond the Right Answer

Marian Small

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Building a perspective

- It is in the elementary grades where students decide what math is all about.

Building a perspective

- Is it about answering lots of questions?
- Should you know the answers right away? Do you have to be fast to be good at math?
- Is there room for doing things your own way or is there a "right" way?

Building a perspective

- Is math about things you write or things you say or both?
- Is using manipulatives something everyone should do or only if you need help?
- Is math something you should do alone or with other people?

Building a perspective

- Are questions welcome or am I supposed to just know what to do?
- Is math easy or hard?
- Is math ever fun?

Building a perspective

- If a child decides, early on, that they are not a "math kid", it can actually doom them to failure.
- We cannot afford not to help young children create a positive mindset.

Building a perspective

- I believe that part of the problems teachers face in teaching math is their own lack of experience with teachers who were comfortable enough in math to veer away from "but this is how you're supposed to do it".

Building a perspective

- So our first order of business is to ensure students see that math is a place where they fit.

Next order of business

- We've talked about students needing to fit into the math world, but what is the math world?
- It may not be as obvious as you think.

The math to focus on

- The curriculum tells you what you are supposed to teach, but A LOT of information is not actually there.

The math to focus on

- Each outcome requires "interpretation".
- Teachers have to decide where exactly the focus should be.

The math to focus on

- For example… Grade 1 Measurement
- Identify common attributes, such as length, height…. that could be used to compare two given objects
- Order a set of objects by length, height… and explain their ordering

The math to focus on

- Determine which of two or more objects is longest or shortest by matching, and explain the reasoning.

The math to focus on

- I could use more "traditional" tasks and simply show students how to compare and have them copy me a number of times.

The math to focus on

- But, instead, I could focus more on their ideas.

So a task might be

- How would you decide if your foot is longer or if your hand is?
- How would you decide if people with longer feet always or usually or only sometimes have longer hands?

So a task might be

- How could you compare the sizes of two pool noodles?
- Could one noodle be bigger one way, but smaller another way?

So a task might be

- One object looks longer than another, but it really isn't.
- What could they be and why would that happen?

So a task might be

- When would it be easy for you to compare the length of two things?
- When would it not be as easy?

Or it might be

- How could you figure out whether the window is taller than how wide the whiteboard is?

Notice

- There is a lot less telling and a lot more asking.

Or it might be Grade 3

- Let's look at fraction understandings.

Content

- Explaining that a fraction represents a part of a whole
- Describing situations in which fractions are used
- Comparing fractions of the same whole with like denominators

Content

- Describing everyday situations where fractions are used
- Cutting or folding a whole into equal parts and naming the parts
- Sorting a given set of shaded regions into those representing equal parts and those that do not, explaining the sorting

Content

- Representing a given fraction concretely or pictorially
- Identifying common characteristics of a given set of fractions

Content

- Naming and recording the fraction represented by shaded and non-shaded parts of a given region
- Identifying numerator and denominator and explaining their meaning

Then we have to make decisions which are not made for us.

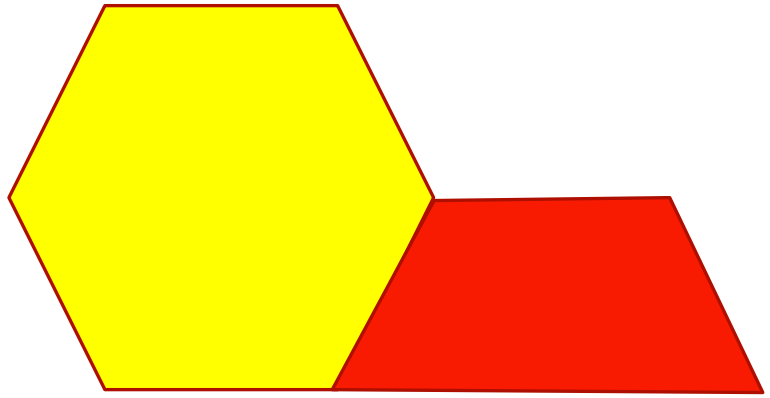
- Should we omit fractions like fifths which rarely, if ever, occur in everyday life or not?
- Do we want fractions of masses or capacities, too, rather than just fractions of area or length?
- Do we focus on exact with pre-divided amounts or do we focus on estimation?

It might be

- Should we be using fractions like 8/12 and relating them to 2/3 yet or not?

It might be

- Do we ask "which is less?" as often as we ask "which is more?"
- Is this half yellow or not?



Let's consider comparing

- Suppose I decide to focus my task on comparing fractions with the same denominator using models.

So my task might be

- **A pizza is cut into 8 slices.**
- **Andy had 3 fewer slices than Brent.**
- What fraction of the pizza could each have?
- Let me give you a minute to think.

So my task might be

- I'd ask:
- Why can't Andy have 3/8 of the pizza?
- Why can't Brent have 8/8?
- Why can't Brent have 3/8 or can he?

Remember: **8 pieces. Andy had 3 fewer slices than Brent.**

Or my task might be

- What fraction value might make sense for this dot? Why?

So my task might be

- And I'd ask:
- Why do you think it's not ½?
- Do you think it could be 2/3?
- What are some other things it could not be?

Let's consider Grade 5 work on division

- Demonstrate, with and without concrete materials, an understanding of division (three-digit by one-digit), and interpret remainders to solve problems.

Content

- Explain why division by 0 is not possible.
- Apply front-end estimation to find quotients
- Investigate a variety of strategies and become proficient in at least one appropriate and efficient division strategy that they understand
- Model the division process as equal sharing, using base ten blocks, and record it symbolically.

Content

- Explain that the interpretation of a remainder depends on the context:

-ignore the remainder

-round up the quotient

-express remainders as a fraction or decimal

Content

- Solve a given division problem in context, using personal strategies, and record the process.
- Refine personal strategies to increase their efficiency
- Create and solve a division problem and record the process

So you will need to decide, e.g.

- Efficient to whom?
- How do we decide what is efficient?
- I am supposed to model equal sharing with base ten blocks, but what should I use to model quotative division?
- Should I usually have remainder problems or only sometimes?

So you will need to decide, e.g.

- What does it really mean anyway to "understand" division?
- Do I need students to explain to me why division is connected to fractions?

Let's look at

- Estimating quotients

Rather than

- Simply giving students quotients and asking for estimates, instead, I might ask…

Possible Task

- I divided two numbers and estimated the quotient might be close to 50.
- What might I have been dividing?

Estimated quotient of 50

- $50 \div 1$
- $52 \div 1$
- $517 \div 10$
- $152 \div 3$

Now

- I can talk about whether the second number (what I divide by) is greater when the first number is.
- E.g. 517 ÷ 10 vs 532 ÷ 10
- E.g. 517 ÷ 10 vs 552 ÷ 11

Now

- I can talk about about whether the estimate changes more if I change the dividend or the divisor.

Now

- I can talk about the number of digits in the dividend and in the divisor and whether I could have predicted the relationship.

Or I could ask

- I divided a 3-digit number by a 2-digit number and my estimate was 20.
- What might I have been dividing?

Quotient estimate of 20

- 400 ÷ 20
- 403 ÷ 20
- 816 ÷ 40
- 625 ÷ 31

And again

- There is a lot to talk about.
- For example, if I get an answer, how can I easily get another one?

And again

- I could increase the dividend by not too much.
- OR I could double both numbers.
- OR..

Possible Task

- You are dividing 729 by 26.
- To estimate, Aliya said it's about the same as 700 ÷ 28 and that's the same as 100 ÷ 4.
- Do you think she's right?
- Is that a good way to estimate?

Again

- I hope you are seeing that the grade level is not the issue.
- At any level, I can focus the conversation more on making sense of what's going on and less on how you do it.

Critical thinking & Creativity

- An important piece of math is the development of critical and creative thinking.

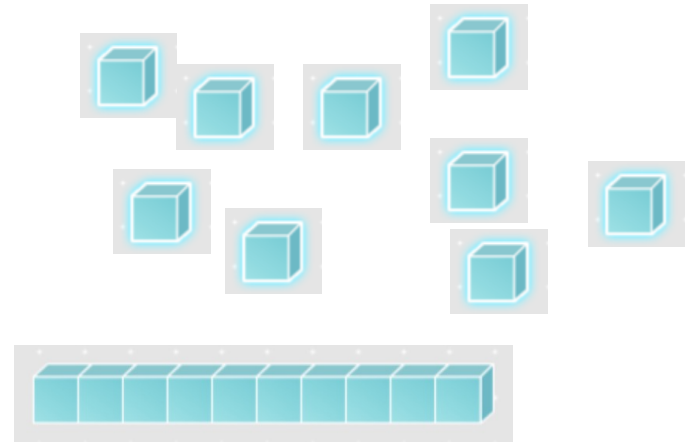
Critical thinking & Creativity

- There are two plates of cookies.
- One has a LOT more cookies than the other.
- How many might be on each plate?

Critical thinking & Creativity

- You show a number with a LOT fewer ten rods than one cubes.
- What could the number be?

Critical thinking & Creativity



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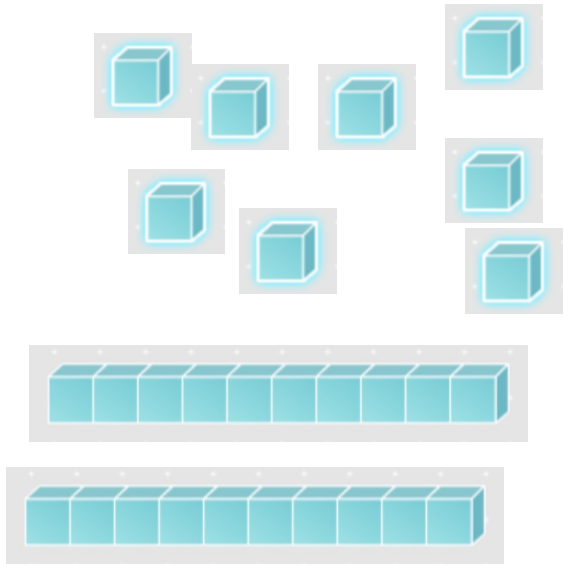

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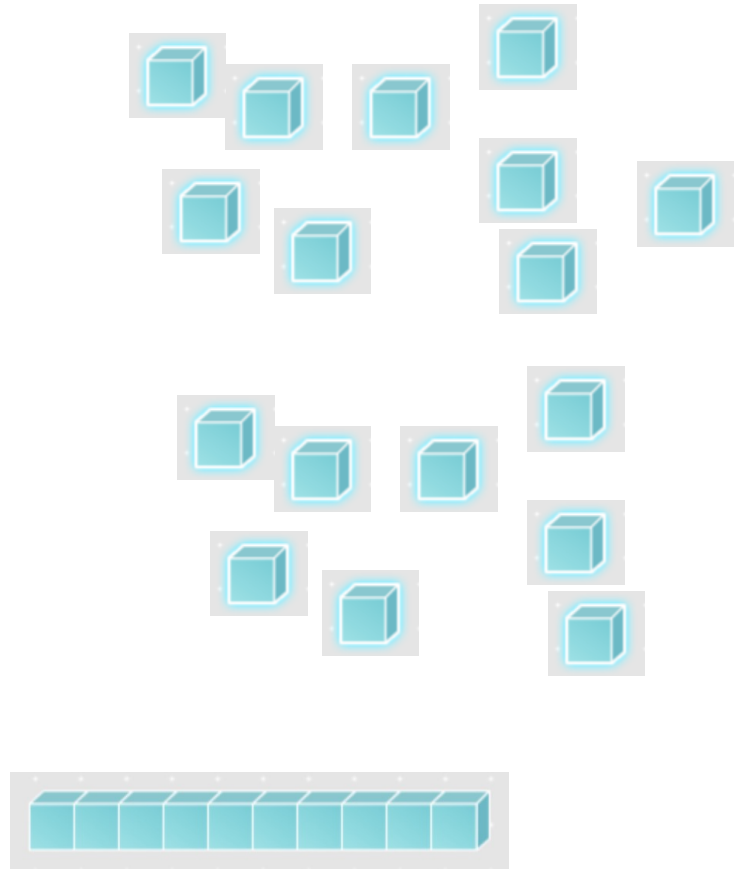
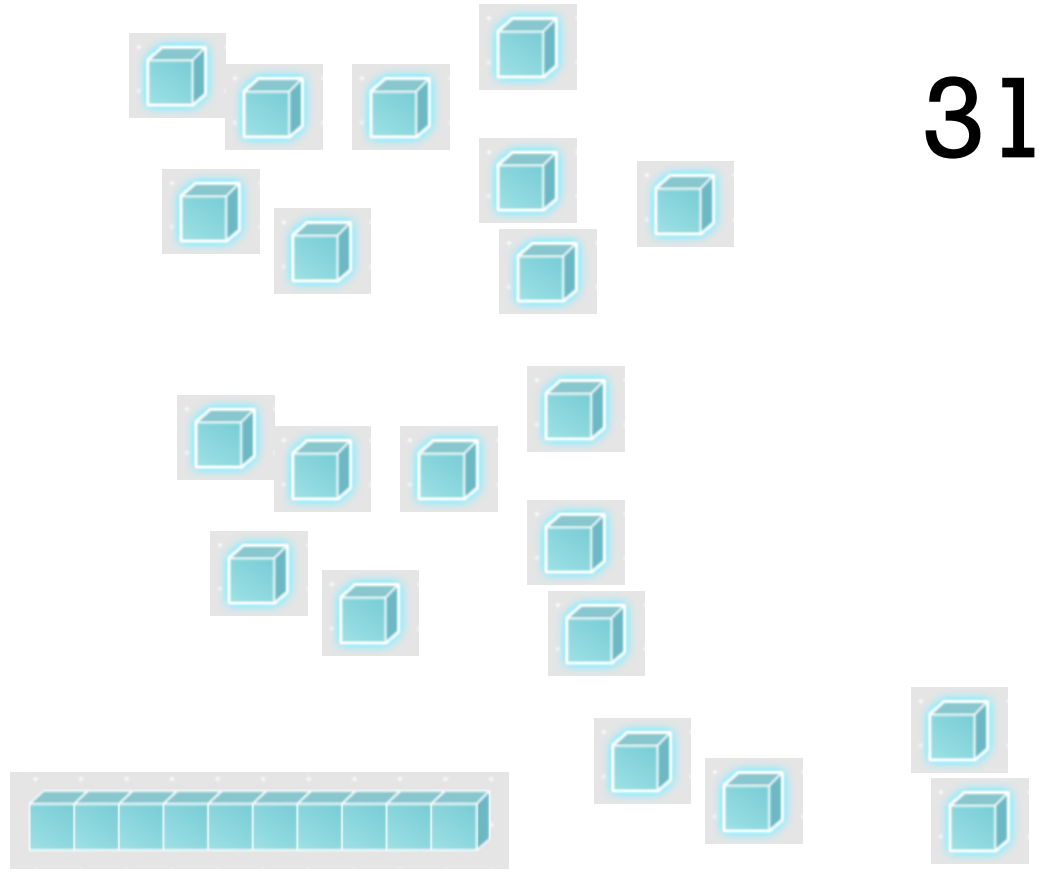
Critical thinking & Creativity



Critical thinking & Creativity



Critical thinking & Creativity

- An amount you can show with 6 coins is added to an amount you can show with 3 coins.
- How many coins might you NEED to show the sum?

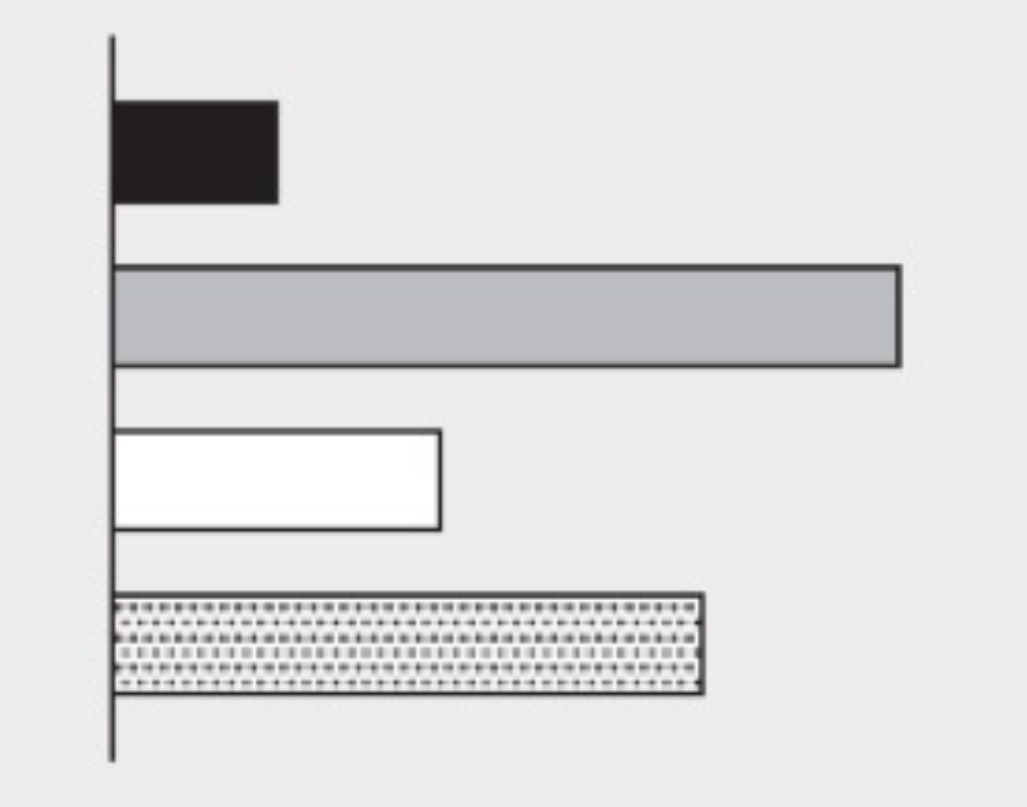
Critical thinking & Creativity

- 6 coins and 3 coins
- QQQQQQ + NNN could be LQQDN
- NNNNNN + DDD could be QQD

Critical thinking & Creativity

- Which is a better description of how far away your birthday is?
- Is it 50 days?
- Is it about 7 weeks?
- Is it almost 2 months?
- Why is it better?

Critical thinking & Creativity



Critical thinking & Creativity

- How could 1000 be a lot?
- How could it be a little?

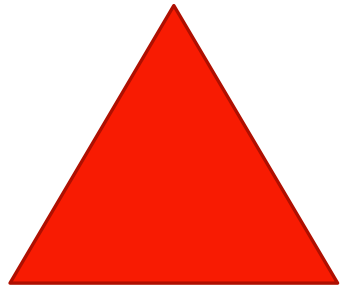
Critical thinking & Creativity

- Which one do you think doesn’t belong? Why?

- 11
- 14
- 23
- 17

Critical thinking & Creativity

- What shape do you think is a lot like this one?



Critical thinking & Creativity

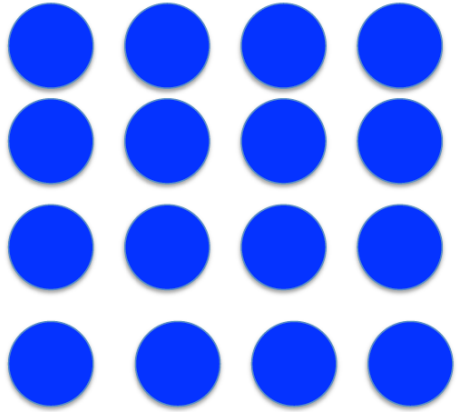
- Is 10% a lot or not?

Critical thinking & Creativity

- What could you measure about an apple?

Critical thinking & Creativity

- What does this tell you about 16?



Critical thinking & Creativity

- Which is probably more?

3.[]4 or [].36

Critical thinking & Creativity

One height is $\frac{7}{8}$ of another.
One height is $1\frac{1}{3}$ times another.
Which is which?

Making Students successful

- We know that kids turn off of math once they can't seem to do what the teacher wants.
- But sometimes they are just not ready for that particular question and we, as teachers, could have avoided the turn-off.

Ensuring the task is appropriate

- The strategy I count on the most is the use of open questions.

Open questions

- The answer is 100.
- What might the question have been?

Maybe

- How old is really old?
- What is the first 3-digit number?
- What is 10 tens?
- What comes after 99?
- What is a perfect score on a test?

More examples

- There are a LOT of people in a car. How many might that be?
- There are NOT MANY students in a school. How many might there be?

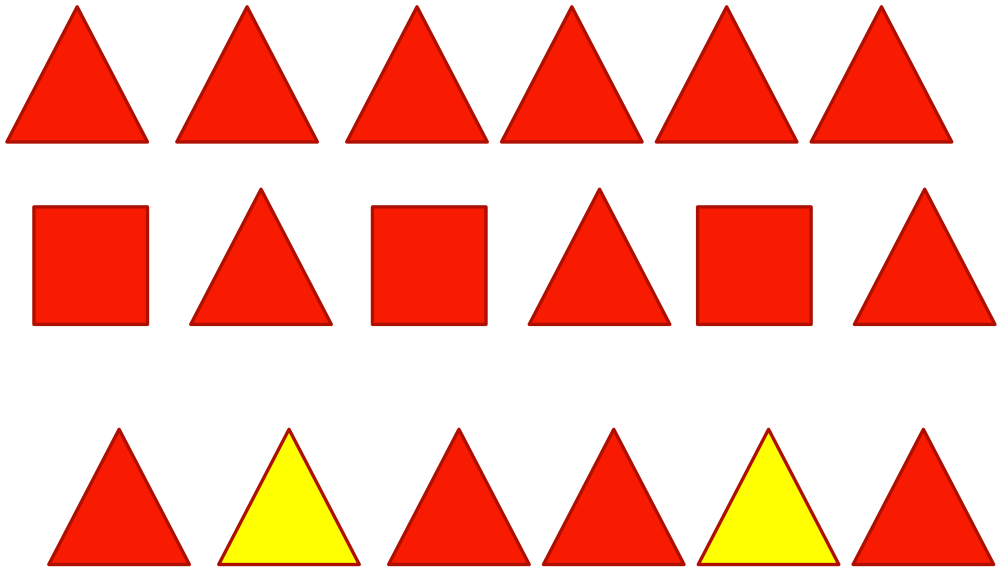
More examples

- How are the numbers 10 and 15 alike? How are they different?

More examples

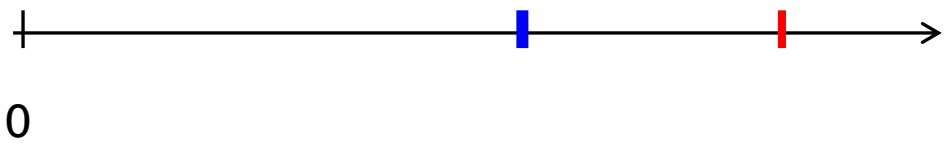
- The 6^{th} shape in a pattern is a red triangle.
- What could the pattern look like?

More examples



More examples

- Choose a number for the red mark. Then choose an appropriate number for the blue mark.



More examples

- A number is just A LITTLE more than ½ .
- What might it be?

More examples

- You buy an item and give the clerk one bill. Your change is one bill and 6 coins.
- What might the price have been?

More examples

- The sum of two numbers is 10 less than the sum of two other numbers.
- What could the four numbers be?

More examples

- The product of two numbers is 10 less than the product of two other numbers.
- What could the four numbers be?

More examples

- You subtract two big numbers and the answer is really small.
- How could that happen?

In summary

- What we need is to move away from:
- Here is how. Now you do it.

In summary

- What we need to move toward is :
- How would you show me….

What you've seen

- is happening in many classrooms now, but perhaps not enough.